A general theory of thermo-compositional adiabatic and diabatic convection

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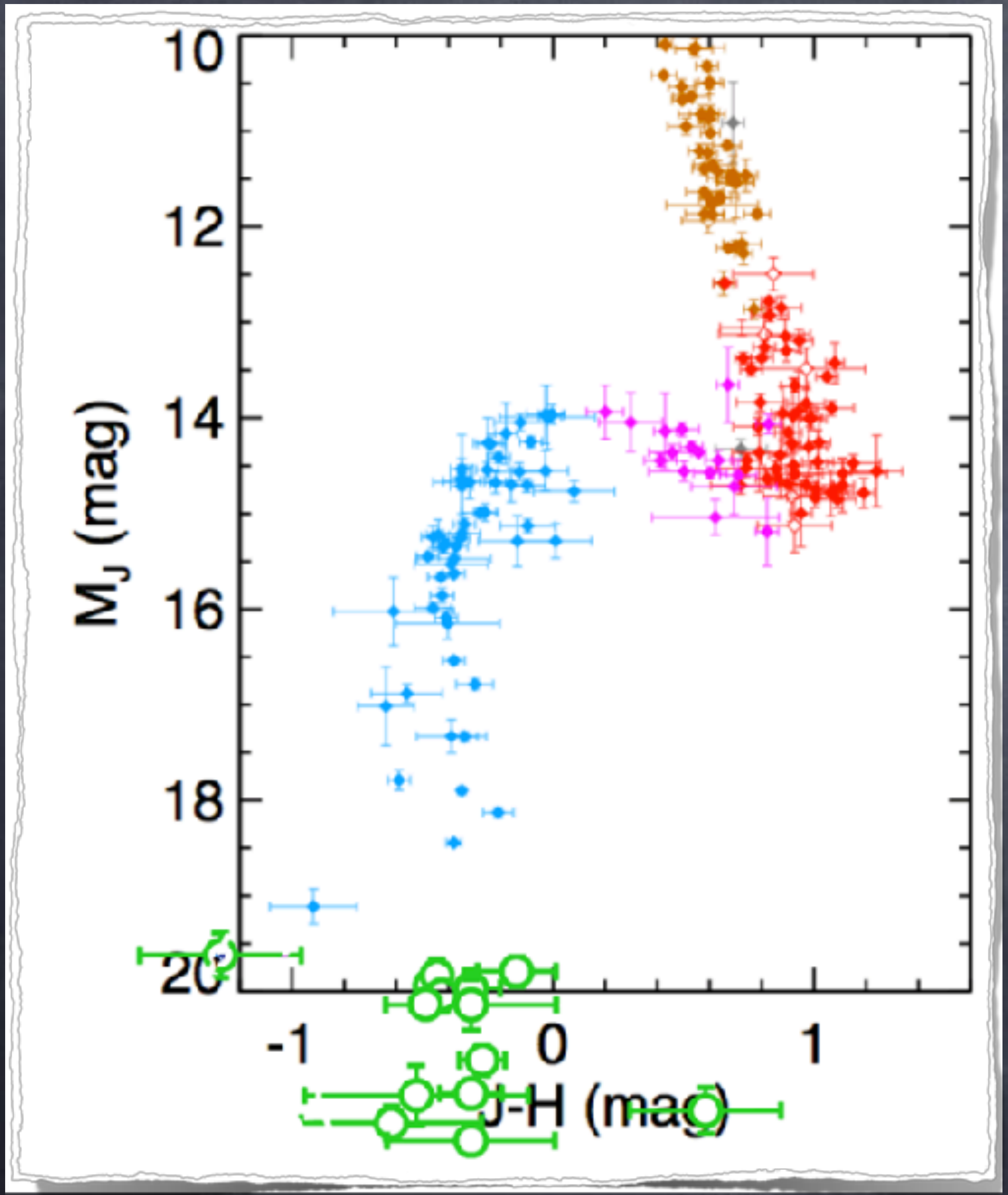
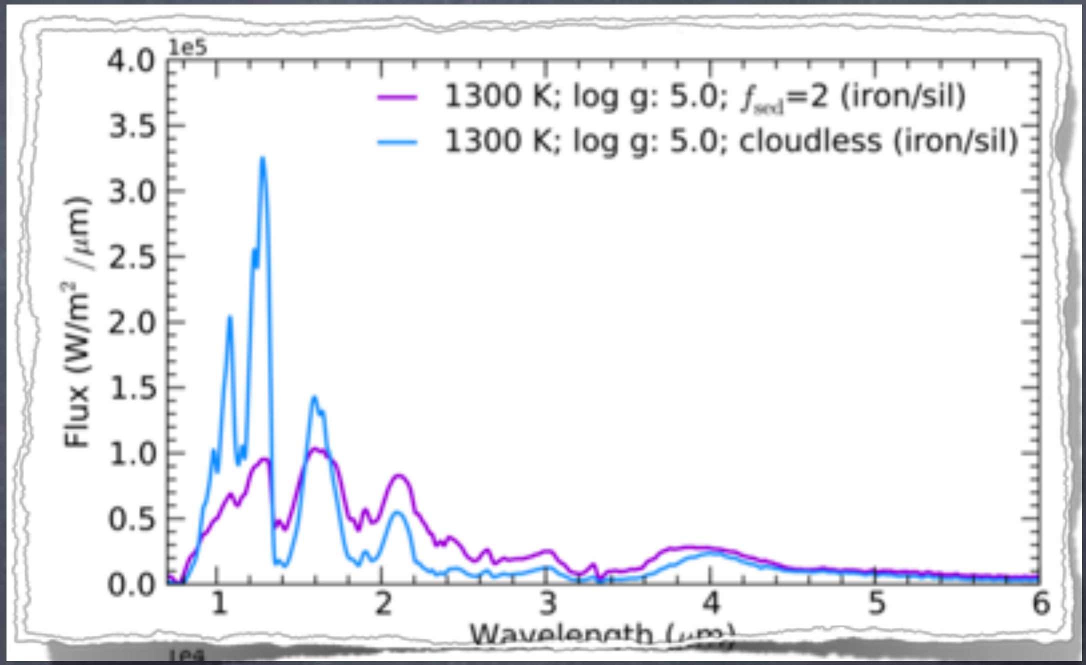
- Brown dwarfs spectral sequence:



Clouds?



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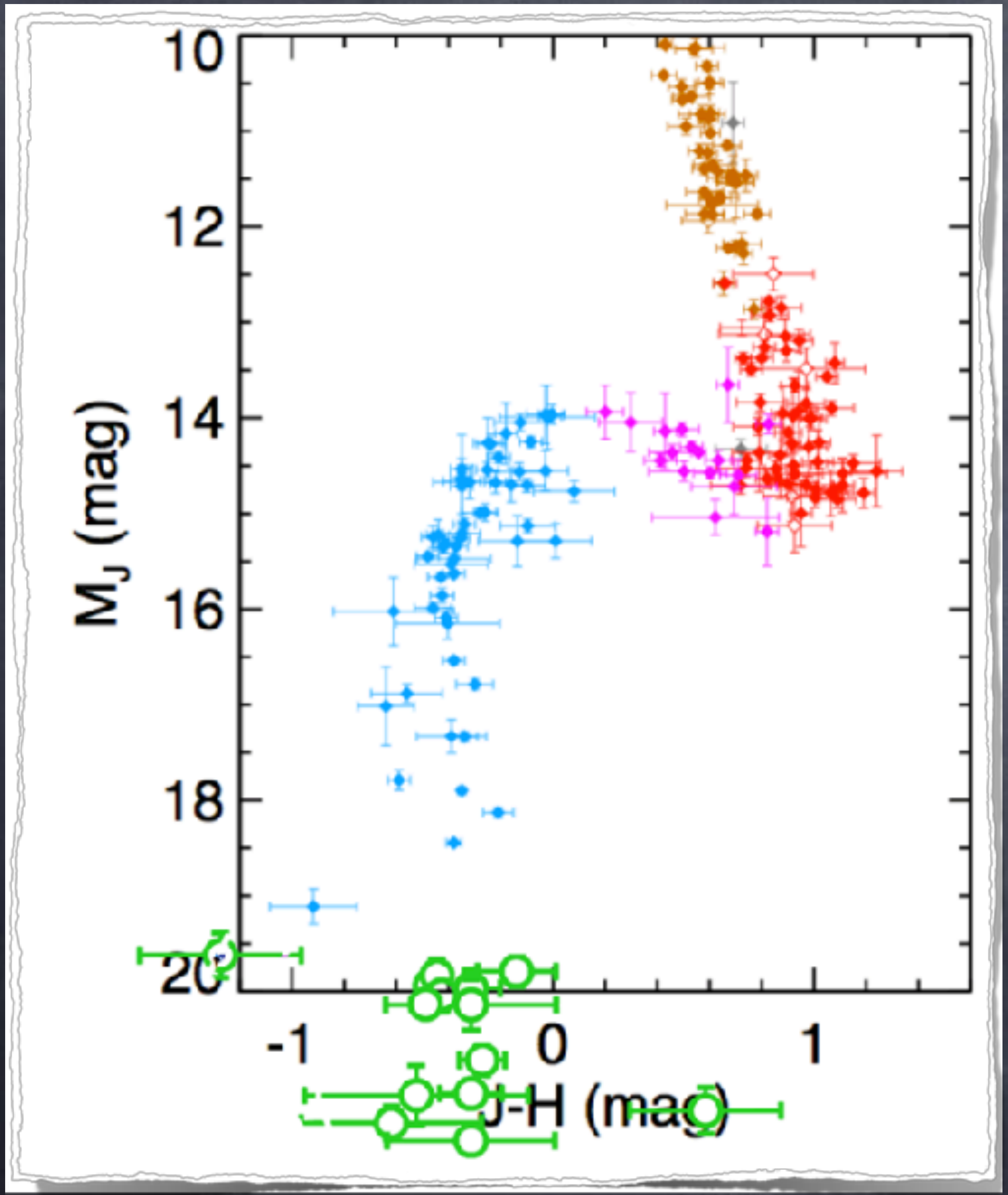
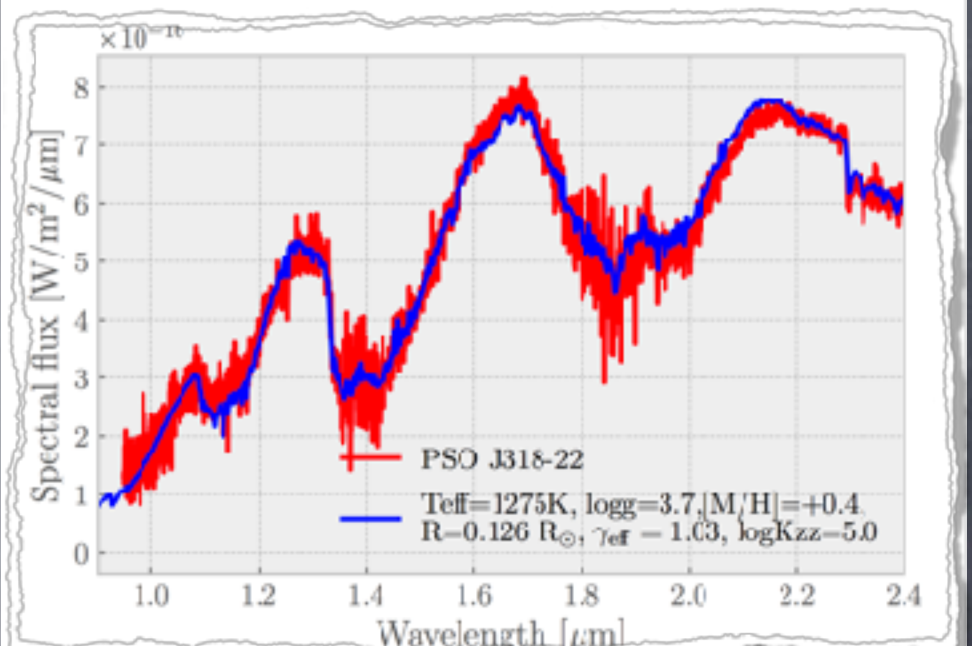
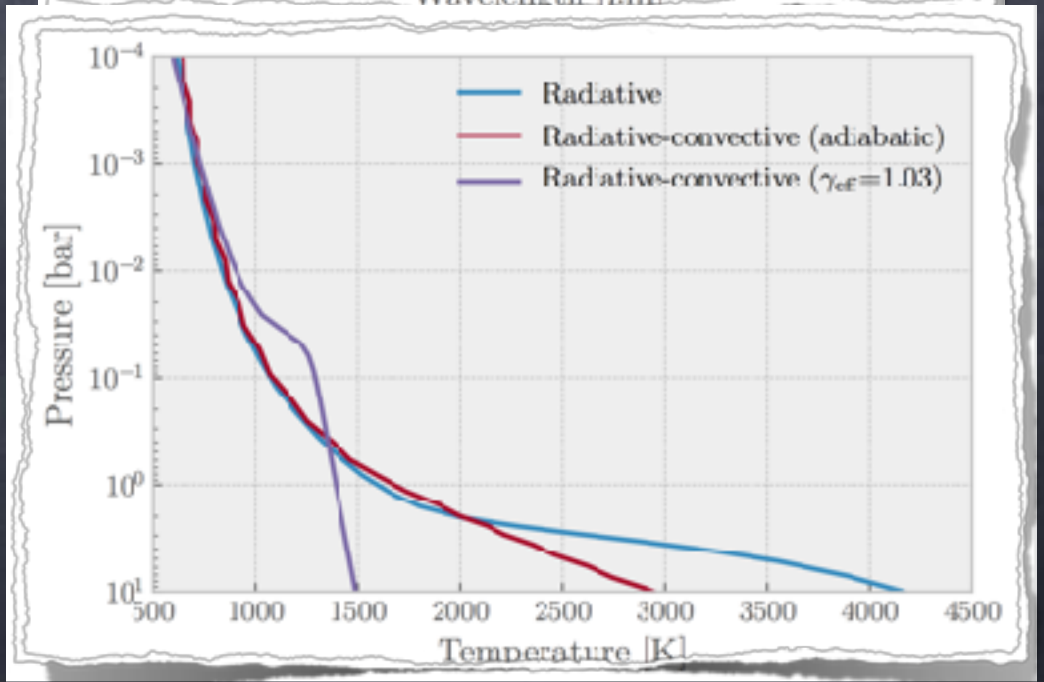


or reduced T gradient?





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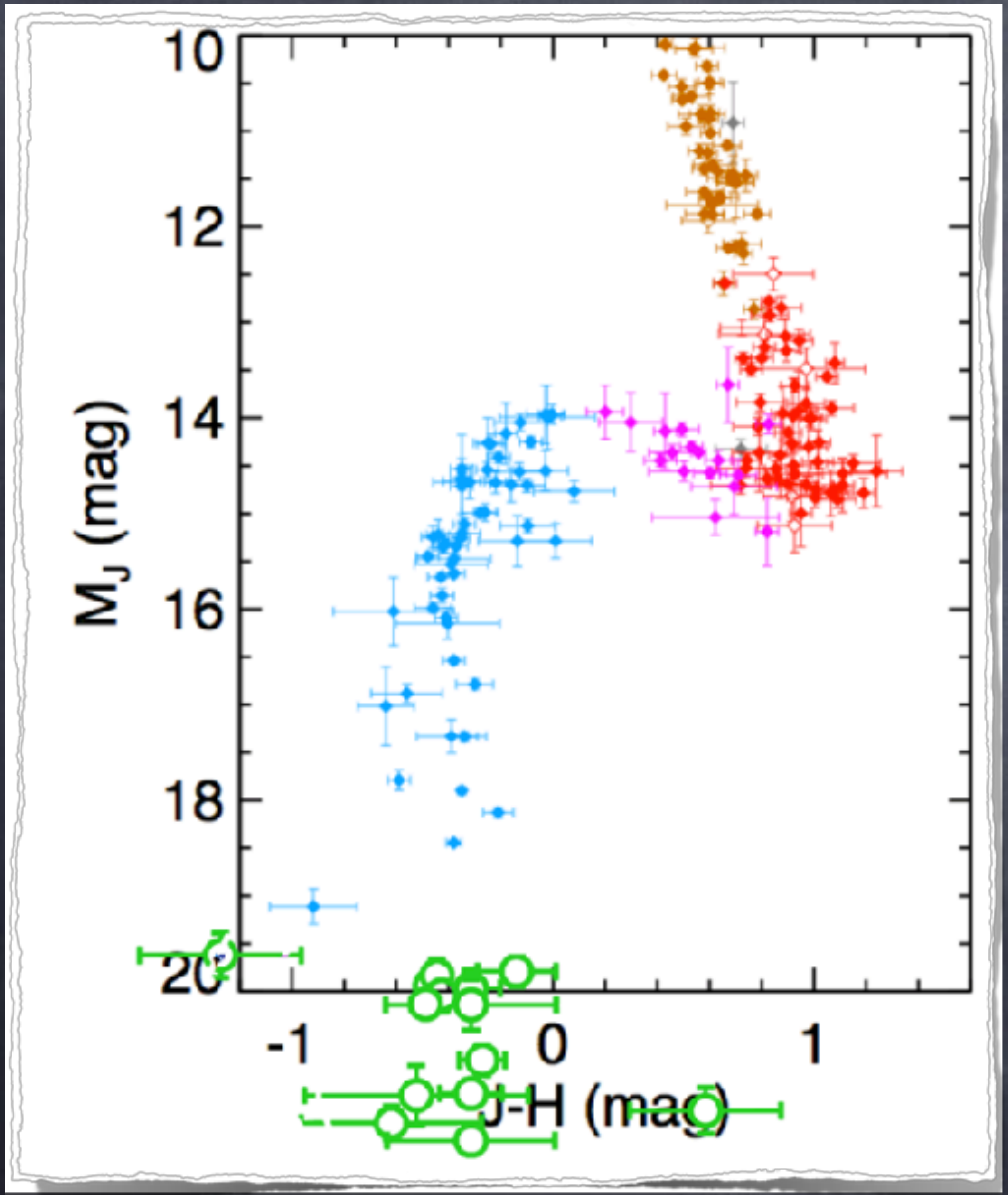

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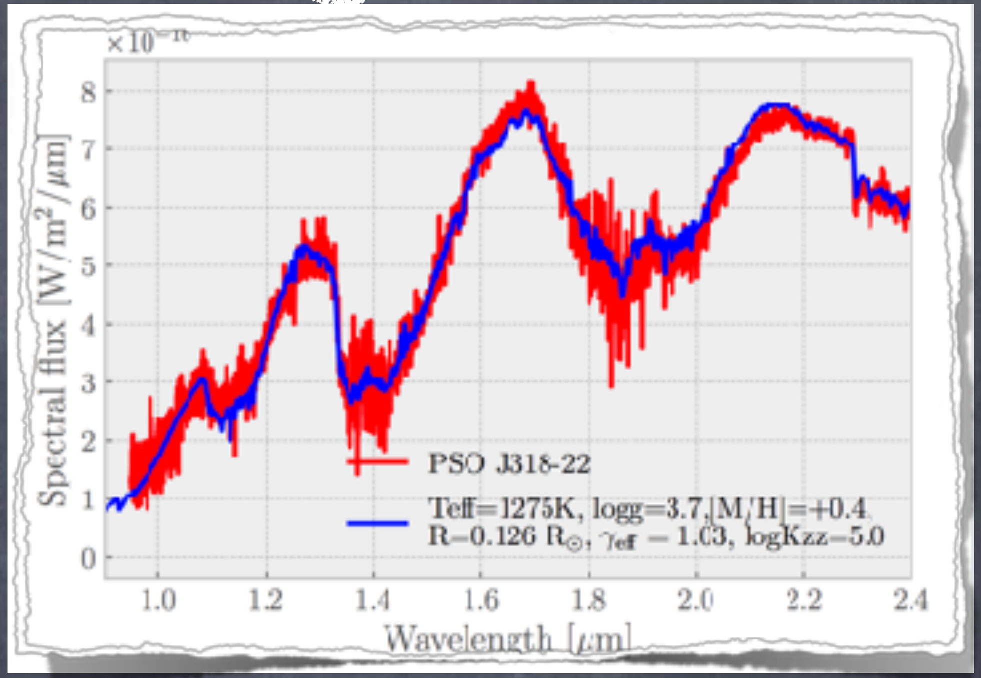
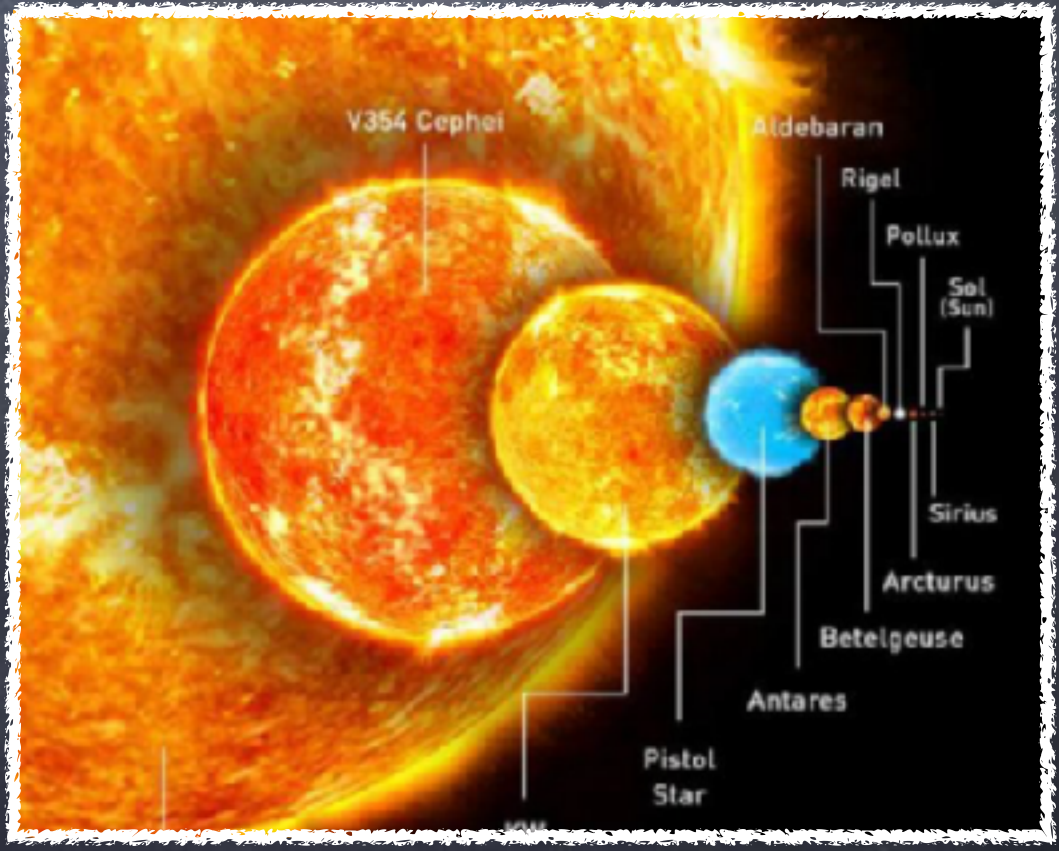
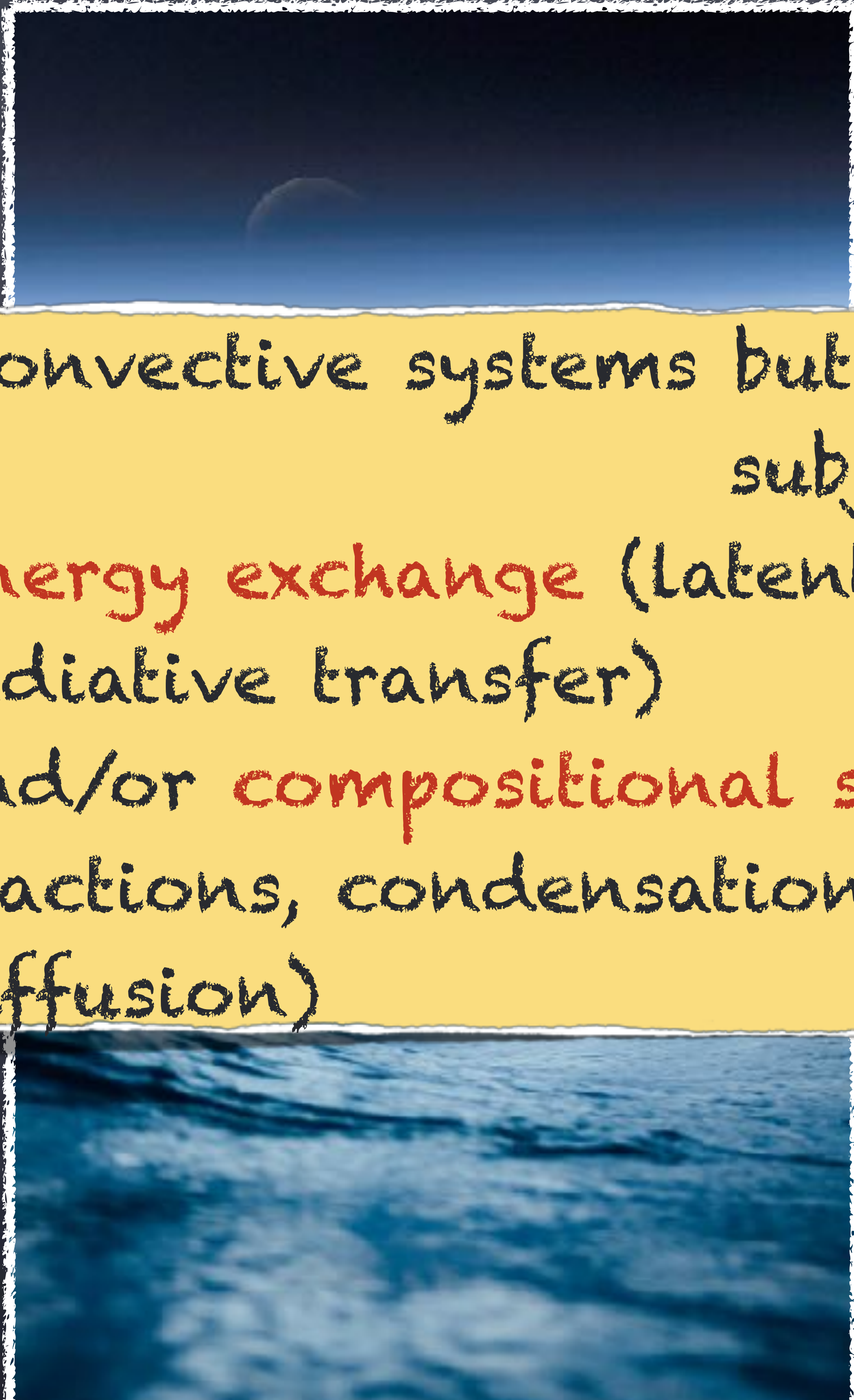


Convection linked to CO/CH4 transition?

- What is in common between:

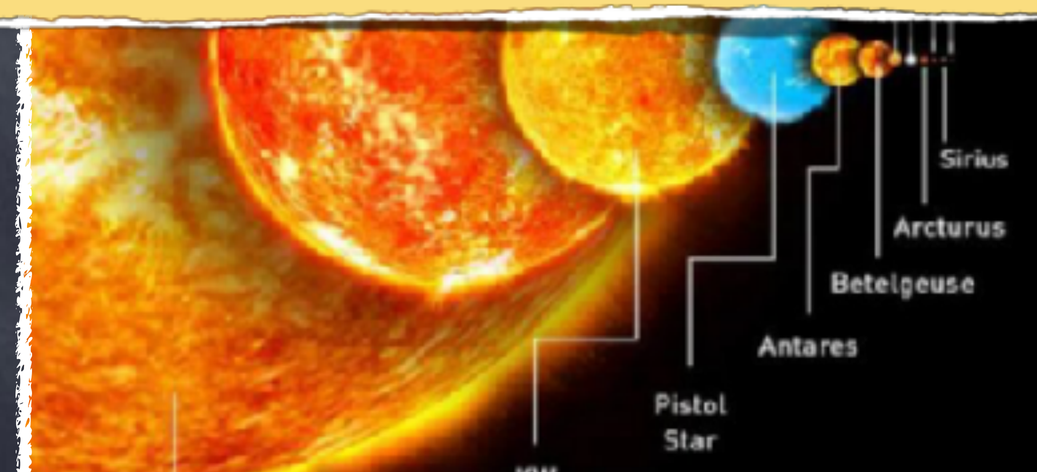


- What is in common between:



Convective systems but not adiabatic, they are all subject to:

- Energy exchange (latent heat, thermal diffusion, radiative transfer)
- and/or compositional source terms (chemical reactions, condensation/evaporation, compositional diffusion)



- What is adiabatic convection?
  - Thermal adiabatic case

$$\frac{\partial \ln \theta}{\partial t} + \overrightarrow{u}\,\overrightarrow{\nabla}\left(\ln \theta\right) = 0$$

$$\theta = T(P_{\mathrm{ref}}/P)^{(\gamma-1)/\gamma}$$

$$P = \rho k_b T/\mu$$

- What is adiabatic convection?
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$$\theta = T(P_{\rm ref}/P)^{(\gamma-1)/\gamma}$$

$$P = \rho k_b T/\mu$$

  - Unstable if: $\frac{\partial \ln \theta_0}{\partial z} < 0$

- Schwarzschild criterion (1906)

$$\nabla_T - \nabla_{\rm ad} > 0, \quad \nabla_T = \frac{\partial \ln T_0}{\partial \ln P_0}$$

$$\frac{\partial T_0}{\partial z} < \frac{g}{C_p}$$

- What is adiabatic convection?
  - Thermo-compositional adiabatic case

$$\frac{\partial \ln \theta}{\partial t} + \overrightarrow{u}\overrightarrow{\nabla}\left(\ln \theta\right) = 0$$

$$\frac{\partial X}{\partial t} + \overrightarrow{u}\overrightarrow{\nabla}(X) = 0$$

$$\theta = T(P_{\mathrm{ref}}/P)^{(\gamma-1)/\gamma}$$

$$P = \rho k_b T/\mu(X)$$

- What is adiabatic convection?
  - Thermo-compositional adiabatic case

$$\frac{\partial \ln \theta}{\partial t} + \overrightarrow{u} \overrightarrow{\nabla} \left(\ln \theta \right) = 0$$

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$$P = \rho k_b T / \mu(X)$$

$$\frac{\partial X}{\partial t} + \overrightarrow{u} \overrightarrow{\nabla} (X) = 0$$

  - Unstable if: $\nabla_T - \nabla_{\mathrm{ad}} - \nabla_\mu > 0$

$$\nabla_T = \frac{\partial \ln T_0}{\partial \ln P_0}, \quad \nabla_\mu = \frac{\partial \ln \mu_0}{\partial \ln P_0}$$

- Ledoux criterion (1947)

- What is adiabatic convection?
  - Thermo-compositional diabatic case

$$\frac{\partial \ln \theta}{\partial t} + \overrightarrow{u}\,\overrightarrow{\nabla}\left(\ln \theta\right) = \frac{H(X,T)}{T}$$

$$\theta = T(P_{\text{ref}}/P)^{(\gamma-1)/\gamma}$$

$$P = \rho k_b T/\mu(X)$$

$$\frac{\partial X}{\partial t} + \overrightarrow{u}\,\overrightarrow{\nabla}\,(X) = R(X,T)$$

- What is adiabatic convection?
  - Thermo-compositional diabatic case

$$\frac{\partial \ln \theta}{\partial t} + \overrightarrow{u}\,\overrightarrow{\nabla}\left(\ln \theta\right) = \frac{H(X,T)}{T} \qquad \theta = T(P_{\mathrm{ref}}/P)^{(\gamma-1)/\gamma}$$

$$\frac{\partial X}{\partial t} + \overrightarrow{u}\,\overrightarrow{\nabla}\,(X) = R(X,T) \qquad P = \rho k_b T/\mu(X)$$

- Unstable if: $\nabla_T - \nabla_{\mathrm{ad}} - \nabla_\mu > 0$

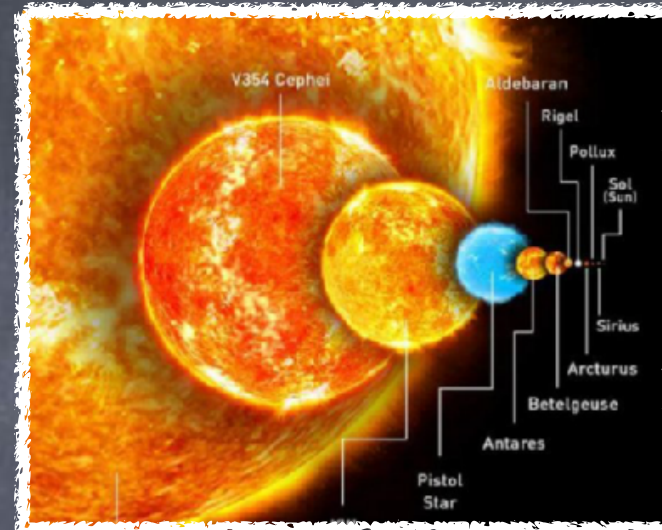
or

$$(\nabla_T - \nabla_{\mathrm{ad}})\omega'_X - \nabla_\mu \omega'_T < 0$$

with $\omega'_X = R_X + R_T(T_0 \partial \ln \mu_0/\partial X)$

and $\omega'_T = H_T + H_X(T_0 \partial \ln \mu_0/\partial X)^{-1}$

- Thermohaline or fingering convection



$$(\nabla_T - \nabla_{\rm ad})\omega'_X - \nabla_\mu \omega'_T < 0$$

with $R = \kappa_\mu \Delta X \quad \omega'_X = -k^2 \kappa_\mu \quad (R_T = 0)$

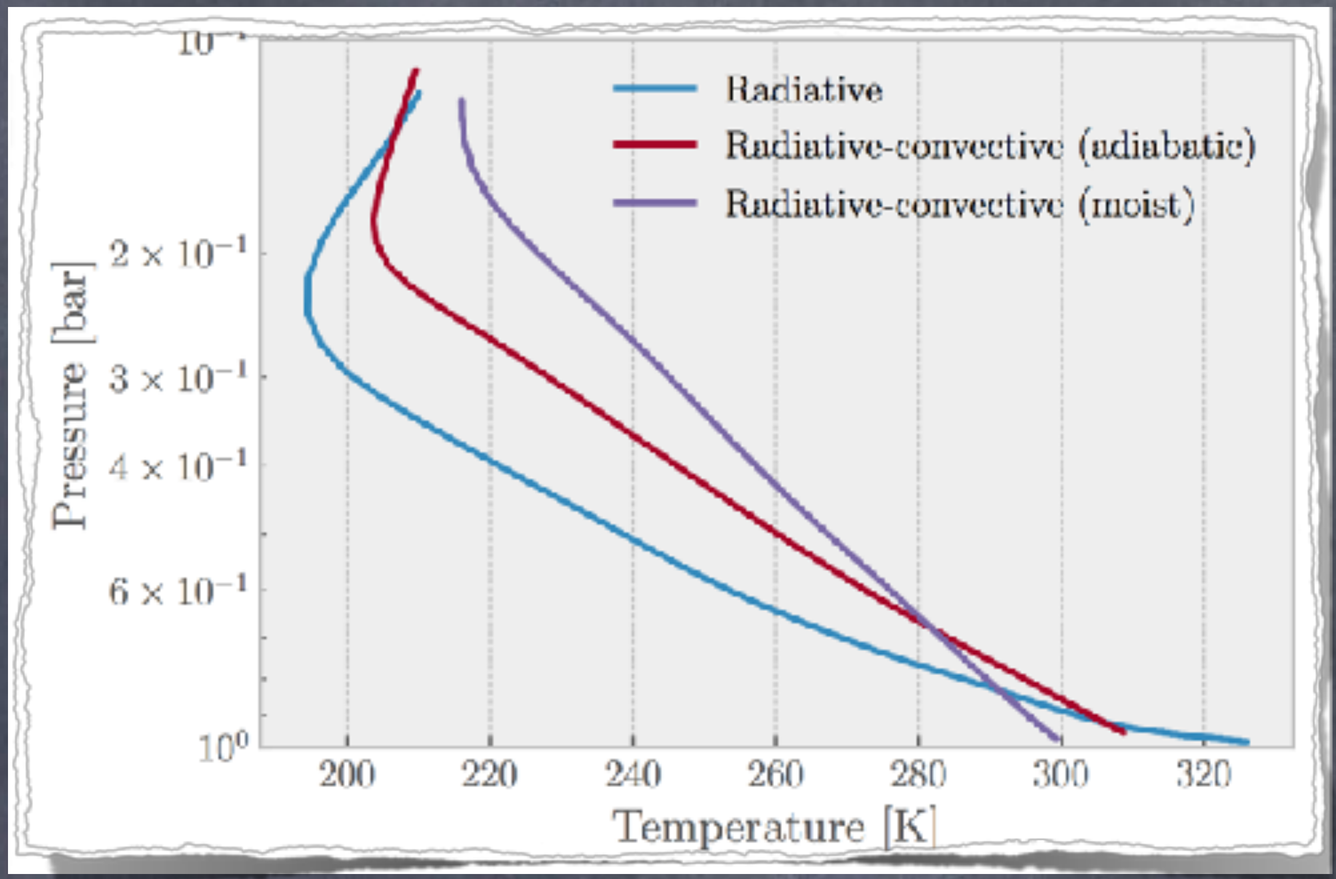
and $H = \kappa_T \Delta T \quad \omega'_T = -k^2 \kappa_T \quad (H_X = 0)$

Stern 1960

Ulrich 1972

$$(\nabla_T - \nabla_{\rm ad})\kappa_\mu - \nabla_\mu \kappa_T > 0$$

- Steam/liquid or moist convection



von Bezold 1893

$$\nabla_T - \nabla_{\mathrm{ad}} > 0$$

Dry adiabat

$$\nabla_T - \nabla_{\mathrm{ad}} \frac{1 + \frac{X_{\mathrm{eq}} L}{R_d T_0}}{1 + \frac{X_{\mathrm{eq}} L^2}{c_p R_v T_0^2}} > 0$$

Moist « pseudo-adiabat »

- Steam/liquid or moist convection

$$(\nabla_T - \nabla_{ad})\omega'_X - \nabla_\mu \omega'_T < 0$$

with $R = R_{cond}(X, T)$ and $X = X_{eq}(P, T)$

and $H = -R_{cond} L / c_p$

von Bezold 1893

$$\nabla_T - \nabla_{ad} \frac{1 - \rho_0 \frac{\partial X_{eq}}{\partial P} L}{1 + \frac{\partial X_{eq}}{\partial T} \frac{L}{c_p}} > 0$$

Moist « pseudo-adiabat »

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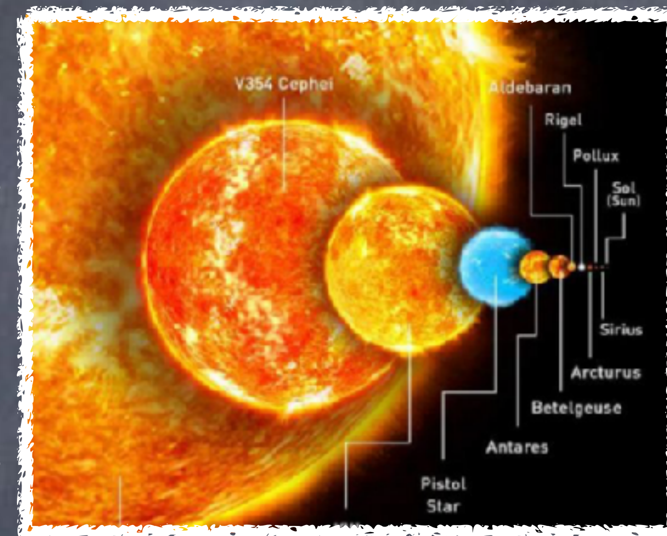
Moist « pseudo-adiabat »

- Thermo-compostional diabatic convection

Moist

Steam/liquid



Fingering

Thermohaline

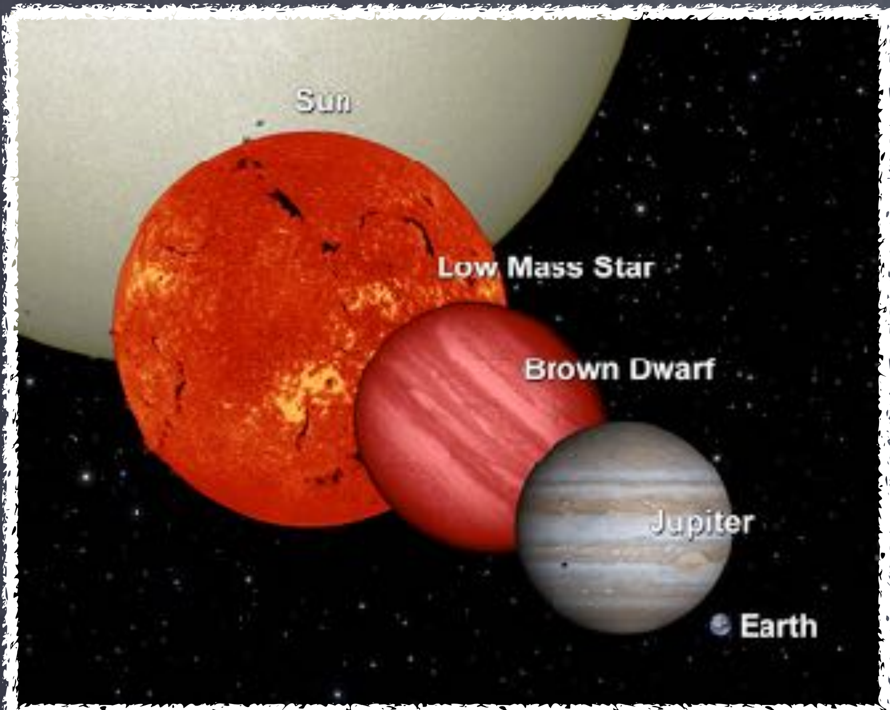
$$(\nabla_T - \nabla_{\rm ad})\omega'_X - \nabla_\mu \omega'_T < 0$$

with $\omega'_X = R_X + R_T(T_0 \partial \ln \mu_0 / \partial X)$

and $\omega'_T = H_T + H_X(T_0 \partial \ln \mu_0 / \partial X)^{-1}$

and probably many more...

- CO/CH4 radiative convection

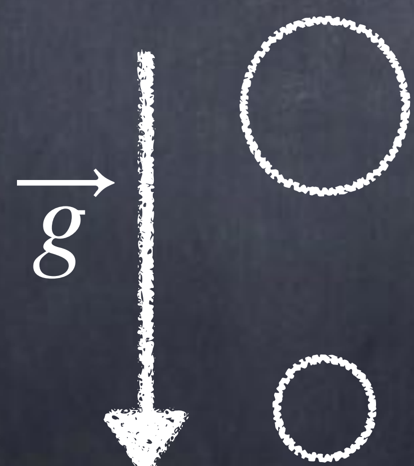


Brown dwarfs and giant exoplanets

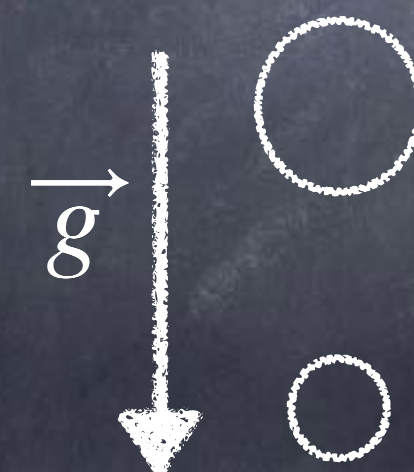
$$(\nabla_T - \nabla_{\rm ad})\omega'_X - \nabla_\mu \omega'_T < 0$$

$$\text{with} \quad R = -(X - X_{\rm eq})/\tau_{\rm chem}$$

$$\text{and} \quad H = 4\pi\kappa/c_p\left(J - \sigma T^4\right)$$



Moist convection



CO/CH4 radiative convection

- Generalisation of mixing length theory

$$\frac{\partial \ln \theta}{\partial t} + \overrightarrow{u}\,\overrightarrow{\nabla}\left(\ln \theta\right) = \omega'_T \frac{\delta T}{T_0}$$

$$\frac{\partial X}{\partial t} + \overrightarrow{u}\,\overrightarrow{\nabla}(X) = \omega'_X \delta X$$

$$\delta X \partial \ln \mu_0 / \partial X = \delta T / T_0$$

- Generalisation of mixing length theory

$$\frac{\partial \ln \theta}{\partial t} + \overrightarrow{u}\,\overrightarrow{\nabla}\left(\ln \theta\right) = \omega'_T \frac{\delta T}{T_0}$$

$$\frac{\partial X}{\partial t} + \overrightarrow{u}\,\overrightarrow{\nabla}(X) = \omega'_X \delta X$$

$$\delta X \partial \ln \mu_0 / \partial X = \delta T / T_0$$

$$\frac{\partial \ln \theta'}{\partial t} + \overrightarrow{u}\,\overrightarrow{\nabla}\left(\ln \theta'\right) = 0$$

with $\ln \theta' = \ln \theta - X \dfrac{\partial \ln \mu_0}{\partial X} \dfrac{\omega'_T}{\omega'_X}$

- Generalisation of mixing length theory

$$\frac{\partial \ln \theta}{\partial t} + \overrightarrow{u}\,\overrightarrow{\nabla}\left(\ln \theta\right) = \omega'_T \frac{\delta T}{T_0}$$

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$$\frac{\partial \ln \theta'}{\partial t} + \overrightarrow{u}\,\overrightarrow{\nabla}\left(\ln \theta'\right) = 0$$

with $\quad \ln \theta' = \ln \theta - X \dfrac{\partial \ln \mu_0}{\partial X} \dfrac{\omega'_T}{\omega'_X}$

$\ln \theta' = \ln \theta - XL/c_p T_0$ for moist convection

- Generalisation of mixing length theory

Can define an adiabatic convective flux:

$$F_{\mathrm{ad}} = \rho c_p w_{\mathrm{ad}} T_0 (\nabla_T - \nabla_{\mathrm{ad}})$$

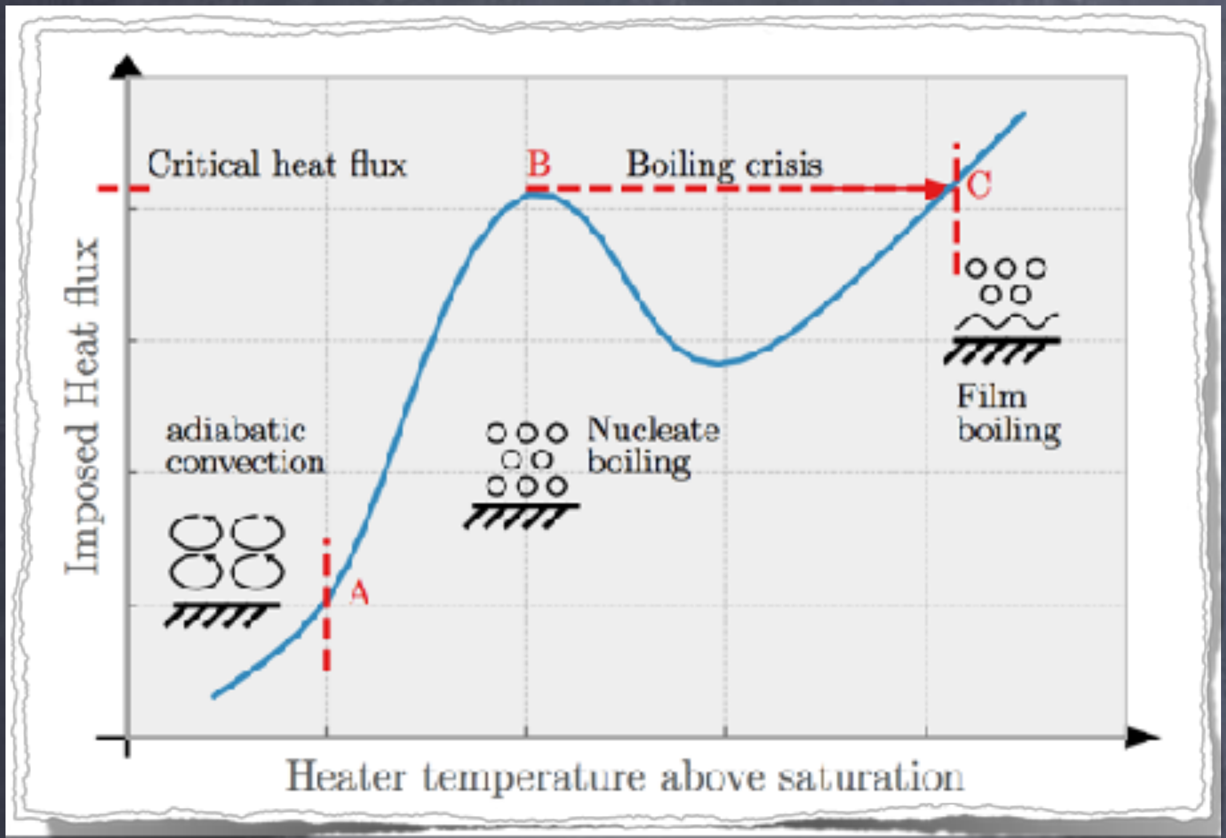
or

Can define a diabatic convective flux:

$$F_{\mathrm{d}} = \rho c_p w_{\mathrm{d}} T_0 (\nabla_T - \nabla_{\mathrm{ad}} - \nabla_\mu \omega'_T / \omega'_X)$$

similar to the mass/flux convection parametrization used for moist convection (Arakawa & Lamb 1981)

- Bifurcation between adiabatic and diabatic convection
  - Boiling crisis in steam/liquid convection



Nukiyama 1934

- Bifurcation between adiabatic and diabatic convection
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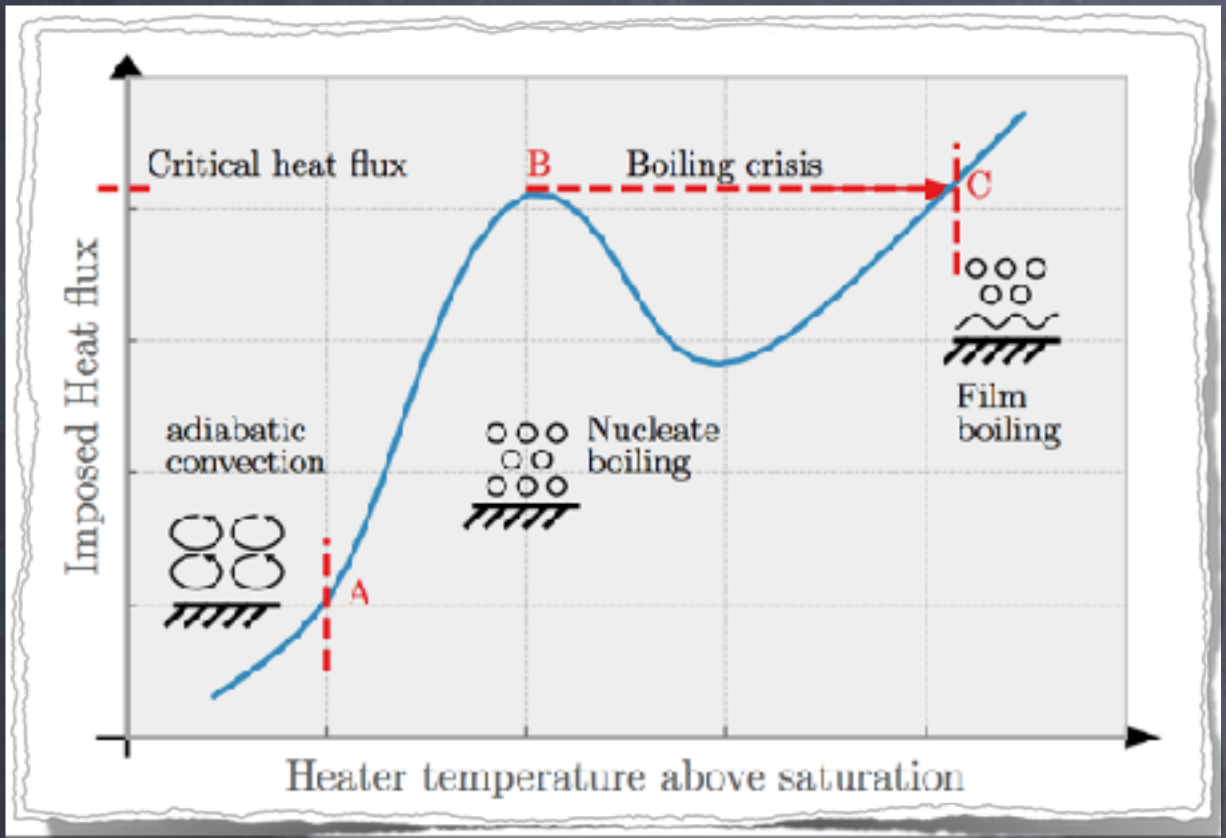
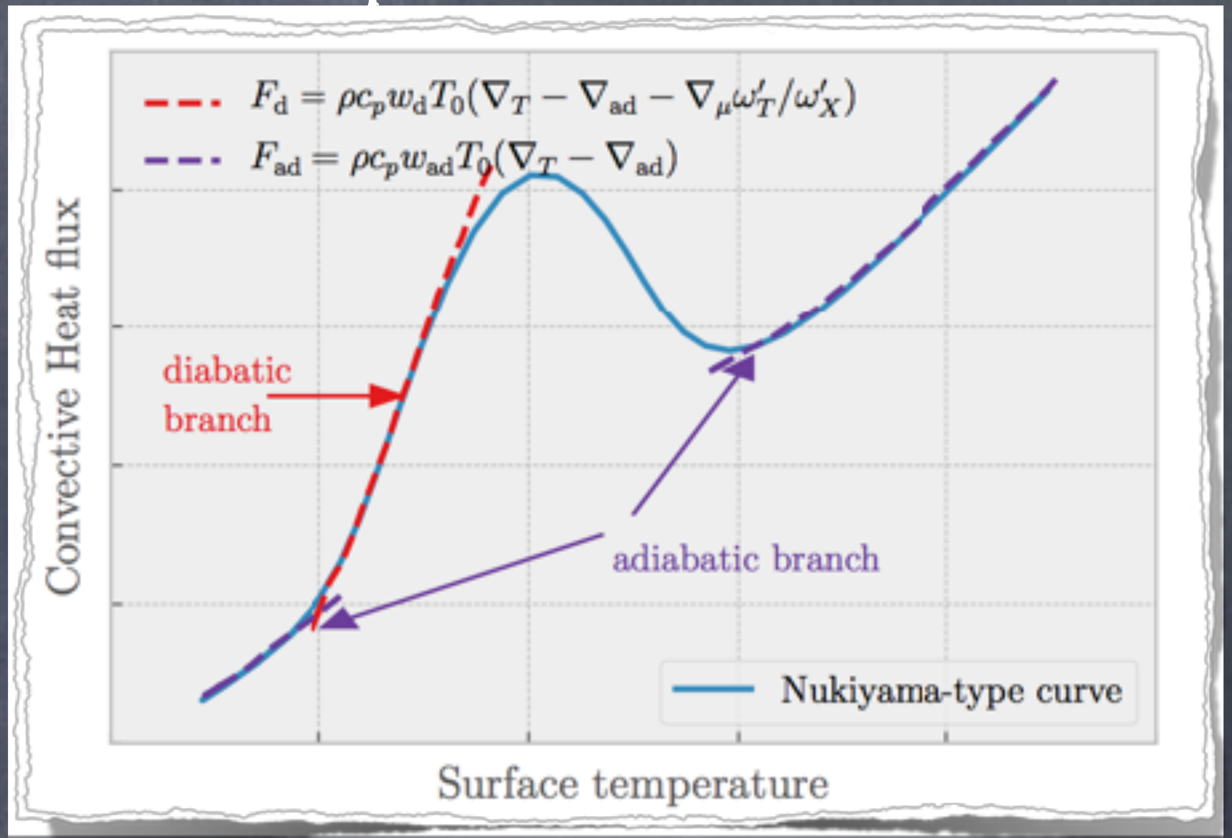
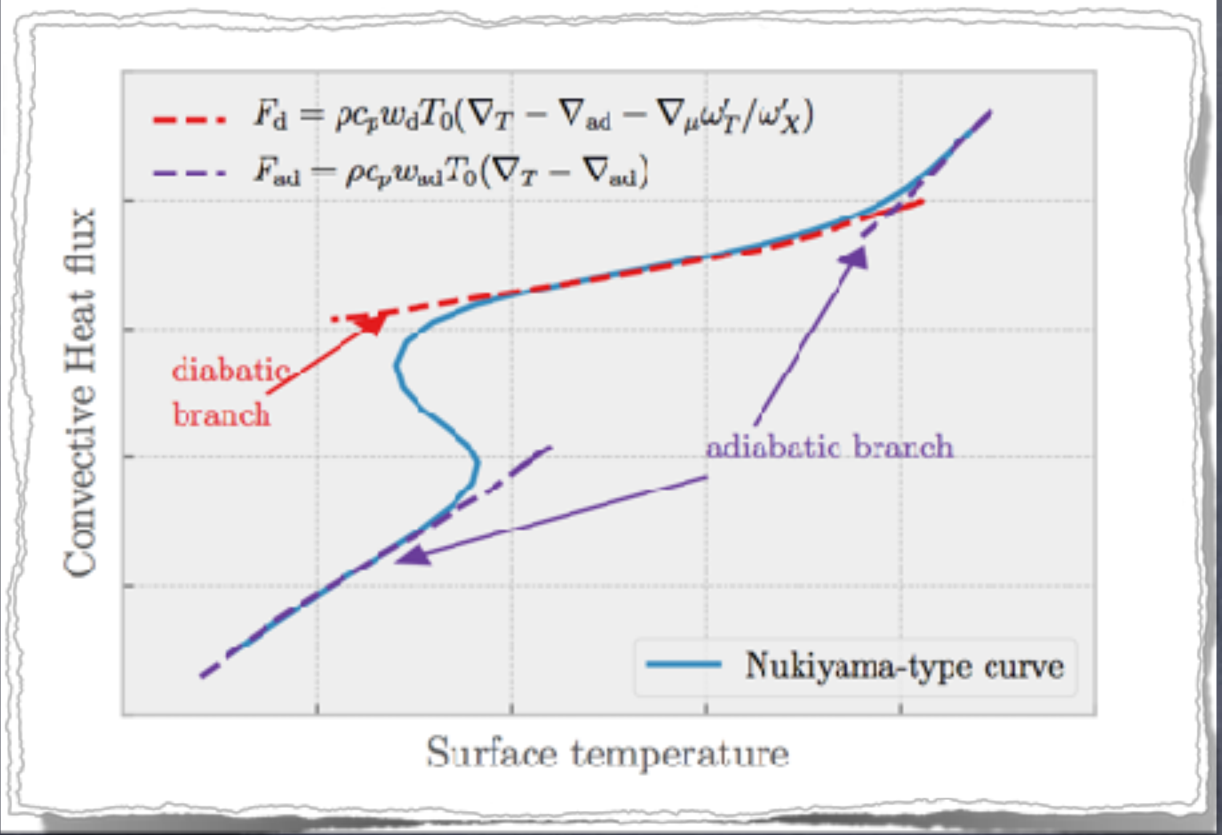


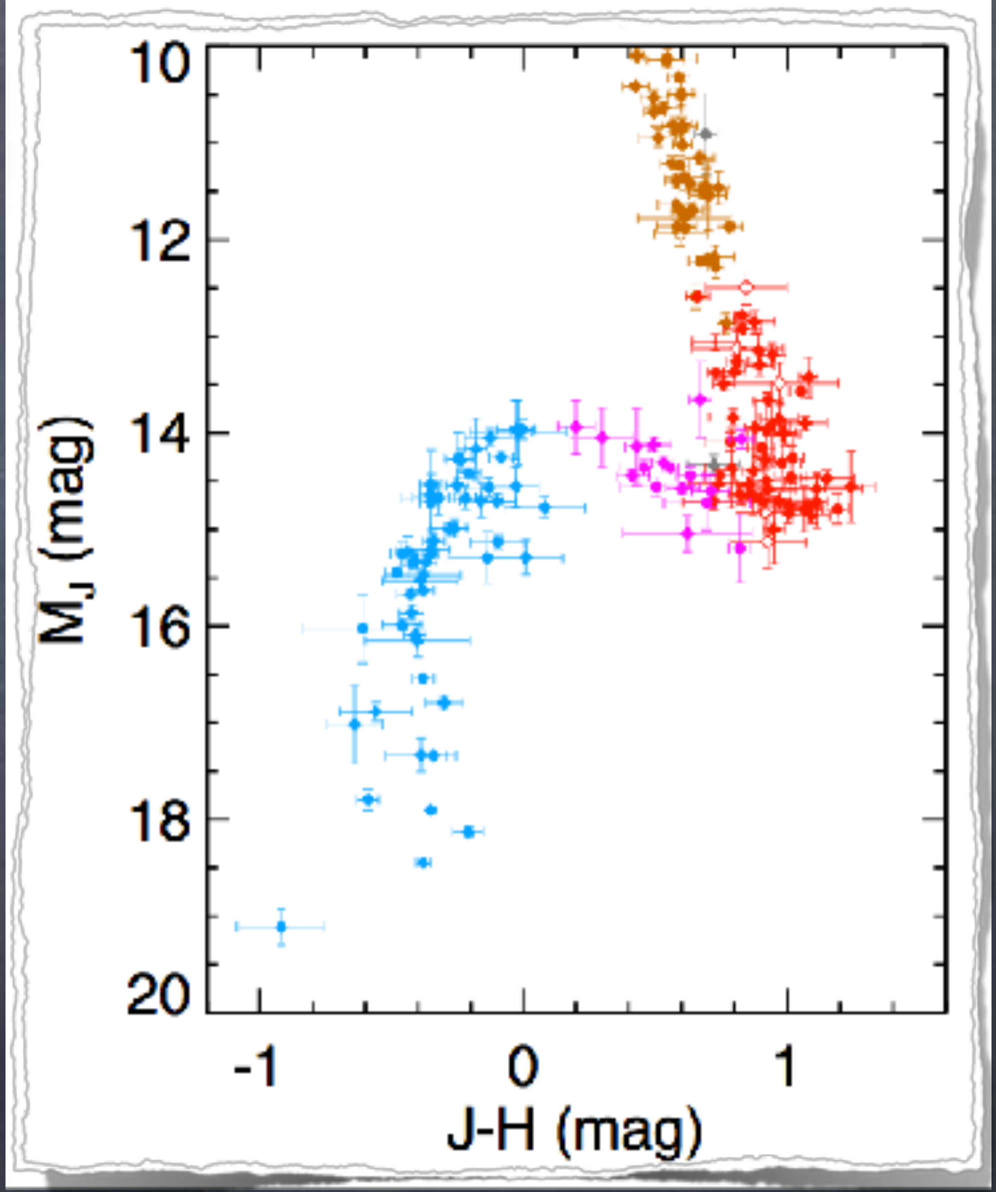


Nukiyama 1934

- Could provide a natural explanation of the boiling crisis?

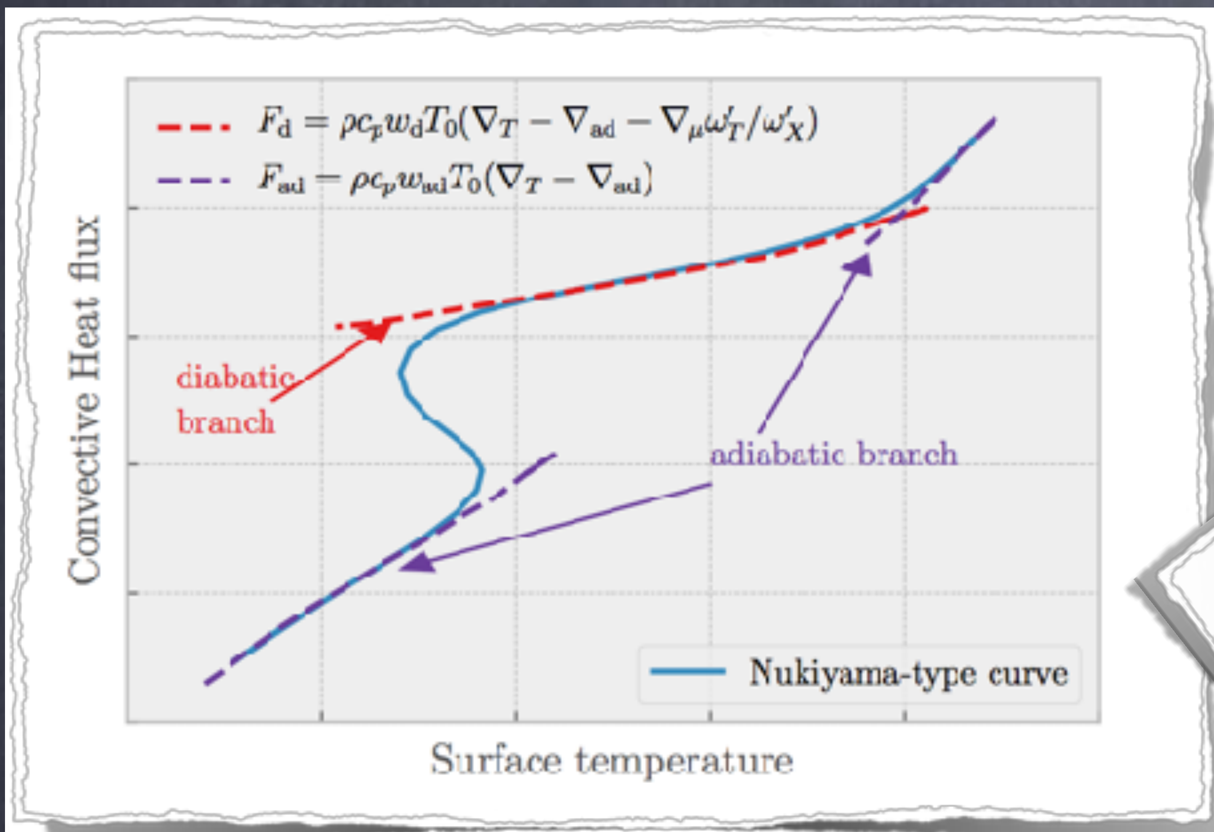
- Bifurcation between adiabatic and diabatic convection
  - L/T transition in brown-dwarf spectra?

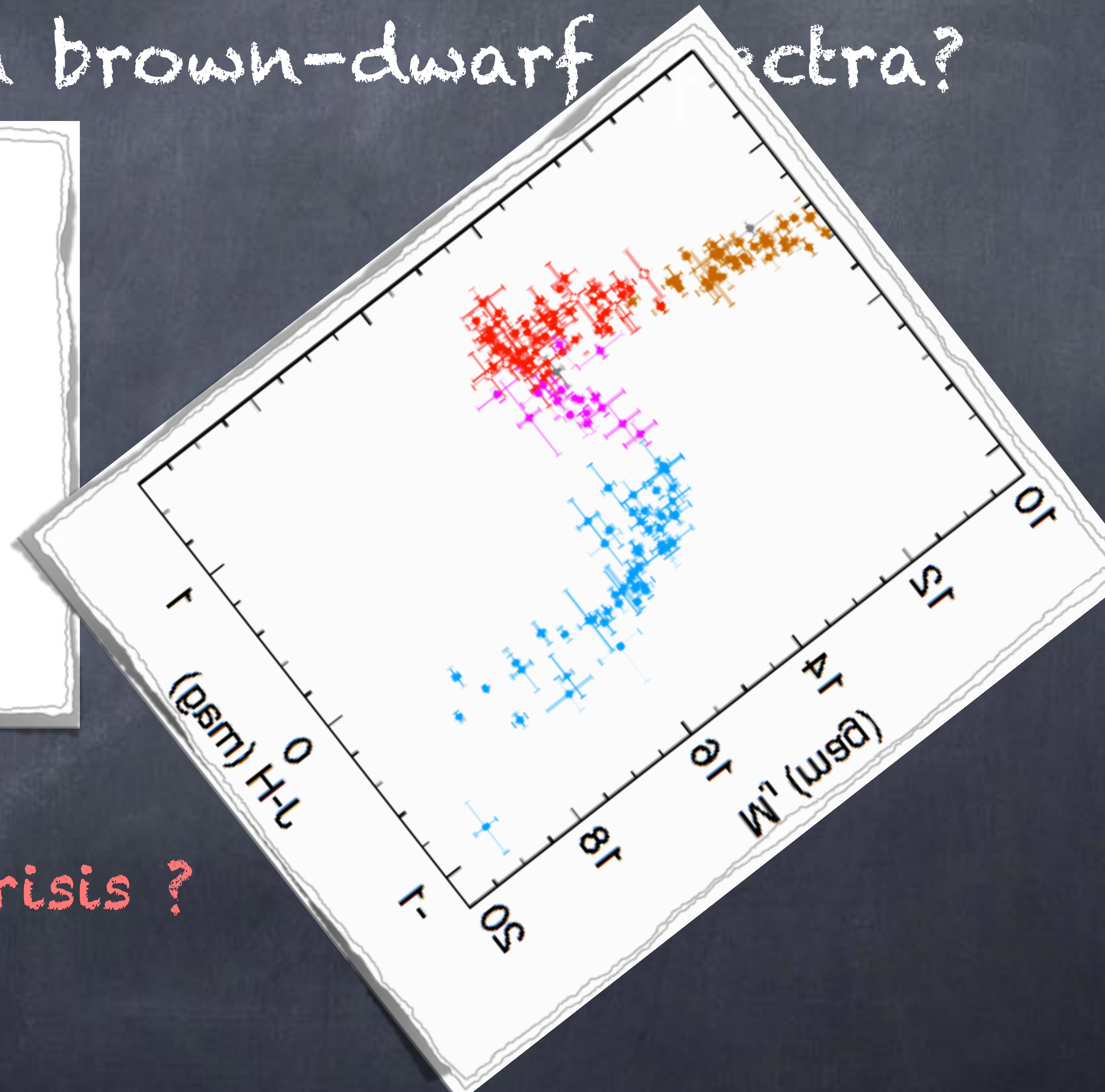




Dupuy & Liu 2012

- Bifurcation between adiabatic and diabatic convection
  - L/T transition in brown-dwarf spectra?





A giant cooling crisis ?